# Skin-Effect Current Distribution of a Unilateral Finline with Finite Conductivity

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## Abstract

This paper presents an accurate full-wave approach to the analyses of current distribution on a unilateral finline containing finite metal conductivity and thickness. The electromagnetic fields are rigorously represented in terms of a complete set of modes, and this allows the classification of the current distributions as edge currents and surface currents contributed by the air modes and the metal modes, respectively. Such an approach provides a clear physical picture for understanding of the cross-sectional current distributions throughout the metal strips, whereby the skin-effect in every direction can be explained. Finally the full-wave analysis establishes a solid basis on which the applicability of the perturbation method can be judged.

## Introduction

Since finlines have become very popular in millimeter-wave integrated circuits, a number of workers[1-3] have extensively analyzed propagation characteristics such as phase constant, characteristic impedance, and loss. Most workers assumed that the metal strips on the finline are perfectly conducting and should be infinitely thin in order to simplify the analysis. In practice, however, the metal strips have finite thickness and conductivity. This is closely related to the complex dispersion characteristics [4] and the power-handling capability [5] of the finline. Both mandate full knowledge of current distributions throughout the metal strips of the finline, including the skin-effect of the surface and edge currents on the surface of the metal as well as the bulk current within the metal. To the authors' knowledge, this important issue has not yet been analyzed and discussed in detail.

Here, we extend the full-wave approach based on the mode-matching method employing a complete set of eigenfunctions describing the electromagnetic field properties of the metal strips with finite conductivity [4]. For the first time, it is shown that skin-effect current distributions can be classified into two categories, namely, edge currents and surface currents contributed by the air modes and the metal modes, respectively. The significance of such classification is at least twofold.

First, the physical insight of the skin-effect currents can be fully exposed by knowing the respective edge and surface current contributions from the air modes and the metal modes. This is achieved by the asymptotic expression of the individual eigenfunction associated with the air modes and the metal modes in the metal strip regions. Section II describes such a procedure briefly. In Section III, numerical data are presented for typical structures and are explained on the basis of asymptotic expressions for the skin-effect currents. Second, based on the exact analytic expressions for the mode functions, the applicability of the perturbation theory for transmission line loss analysis can be determined. Thus, it will become obvious when the amplitude of the surface current near the edge of the metal fin is comparable to that of the edge current and when the perturbation approach can no longer hold.

## The Asymptotic Expressions for the Edge Currents and Surface Currents

Fig. 1 shows the cross-section of a unilateral finline. In treating the symmetrical case of the unilateral finline, the structure can be placed by a perfect electric conductor(PEC)

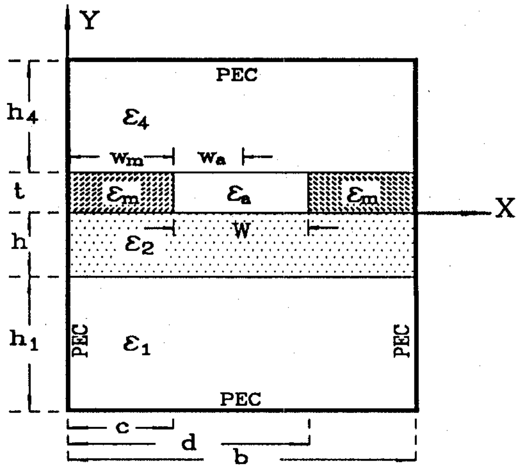


Fig. 1 The symmetrical unilateral finline with finite conductivity and thickness. b=3.556 mm, c=0.889 mm, d=2.667 mm, $h_1$=3.4925 mm, h=0.127 mm, t+$h_4$=3.4925 mm, $\varepsilon_1=\varepsilon_a=\varepsilon_4=1$, $\varepsilon_2=2.22$, $\varepsilon_m=1\text{-}j\sigma/\omega\varepsilon_o$, and $\sigma=3.333\times10^7$ S/m.



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plane at x=b/2 for the odd modes. To employ the mode matching method[4], the structure may be divided into four sub-regions: three uniform regions and one nonuniform layer region containing the metal strip. The eigenvalue equation for the nonuniform layer region is

$$\tan(\kappa_a w_a)\cot(\kappa_m w_m)=-Z_m/Z_a \quad (1)$$

where $w_m+w_a=b/2$, $\kappa_i=k_o(\varepsilon_i-\varepsilon_{eff})^{1/2}$, $Z_i=\omega\mu_o/\kappa_i$ for the TE mode and $Z_i=\kappa_i/\omega\varepsilon_o\varepsilon_i$ for the TM mode. The subscript i may stand for either "a" or "m" to denote the air region or metal region, correspondingly. The aim of this section is to determine the asymptotic expressions for the current densities in the metal strips. A TE mode has its electric field in the y and z directions and is expected to induce a larger current than a TM mode does for a case of high conductivity of metal strips. Although we do employ hybrid modes to analyze the finline structure, the analytic expressions are given here only for TE modes, just to illustrate the underlying mathematical and physical concepts.

### (a) Edge Currents : Contributed by the Air Modes

The electric field of a TE mode is polarized in the z direction. The normalized mode function of an air mode is simply given by

$$\Phi^{(a)}{}_a(x)=\sin(\kappa_a(w_m+w_a-x)),\quad w_m\le x\le w_m+w_a \quad (2a)$$
$$\Phi^{(a)}{}_m(x)=\sin(\kappa_a w_a)\sin(\kappa_m x)/\sin(\kappa_m w_m),\quad 0\le x\le w_m \quad (2b)$$

Invoking (1), we find that for the $n^{th}$ TE mode, the transverse phase constant is given by

$$\kappa_a\approx n\pi/w_a \quad (3)$$

for n=1, 2, 3,... Thus, for a good conductor the electric field $E^{(a)}{}_{zm}(x)$ in the metal region is given approximately by

$$E^{(a)}{}_{zm}(x)=E_{z0}\omega\delta e^{-(1+j)(w_m-x)/\delta} \quad (4)$$

where $E_{z0}$ is an arbitrary constant and $\delta$ is the skin depth. The conduction current density **J** is defined as the product $\sigma$**E**. The edge current associated with an air mode is given by

$$|J^{(a)}{}_{ze}(x)|=J_{z0}e^{-(w_m-x)/\delta}/\delta \quad (5)$$

where $J_{z0}$ is an arbitrary constant, depending on the source of excitation. Evidently, (5) manifests the fact that edge current decays exponentially from the surface at $x=w_m$ into the metal as the skin-effect theory predicts. Similarly, the other tangential component of a current polarized in the y direction can be obtained as follows

$$|J^{(a)}{}_{ye}(x)|=J_{y0}e^{-(w_m-x)/\delta}/\delta \quad (6)$$

where $J_{y0}$ is an arbitrary constant. A small normal component of electric field $E_x$ also exists in the conductor, and it should be derived from the TM mode which is expressed in terms of $\cos(\kappa_a(b/2-x))$ form in contrast to (2a). $J_x$ can be derived as

$$|J^{(a)}{}_{xe}(x)|=J_{x0}\omega e^{-(w_m-x)/\delta} \quad (7)$$

where $J_{x0}$ is an arbitrary constant.

### (b) Surface Currents : Contributed by the Metal Modes

The normalized mode function of a TE metal mode is simply given by:

$$\Phi^{(m)}{}_m(x)=\sin(\kappa_m x),\quad 0\le x\le w_m \quad (8a)$$
$$\Phi^{(m)}{}_a(x)=\sin(\kappa_m w_m)\sin(\kappa_m(w_m+w_a-x))/\sin(\kappa_a w_a),\quad w_m\le x\le w_m+w_a \quad (8b)$$

Invoking (1), the transverse phase constant is given by:

$$\kappa_m\approx n\pi/w_m \quad (9)$$

for n=1, 2, 3, ... For the $n^{th}$ TE metal mode, the electric field in the metal region varies sinusoidally over the surface of the strip (from x=0 to $w_m$), as given by (8a). The total surface current is the superposition of the individual current contributed by all the metal modes as follows

$$|J^{(m)}{}_{zs}(x)|=\Sigma J_{ozn}\omega\sin(n\pi x/w_m) \quad (10)$$

where $J_{ozn}$ is an arbitrary constant, depending on the source of excitation and the structural parameters. The surface currents contribute to the bulk current distributions inside the metal not dealt with before in the literature. Obviously, such surface currents will be responsible for another skin-effect in the direction perpendicular to the fin.

## Results

Given a case with the structural and material parameters shown in Fig. 1, the complex propagation constant of the dominant finline mode can be found rigorously [4]. Here we apply 50 air and metal modes to investigate the edge currents and surface currents, respectively. It is noted that the data presented herein are normalized such that the Poynting power carried by the dominant finline mode is one watt.

Figs. 2(a) and (b) plot the slot-fields ($E_x$ and $E_z$) in the air region ($\varepsilon_a$) and current densities ($J_x$ and $J_z$) in the metal region ($\varepsilon_m$) versus normalized coordinates along the x direction at different locations, $y_1$ and $y_2$, for a particular case where the ratio of metal thickness to the skin depth ($t/\delta$) is 11.5. In (10), the metal modes produce the surface currents computed by including 50 harmonic terms. Both current densities $J_{x2}$ and $J_{z2}$ decay in $e^{-|y_2-y_1|/\delta}$ factor along the y direction except at x=c, where there exist the edge currents and the skin-effect in the y direction is no longer visible. From Figs. 2 (a) and (b), we observe that the electric fields in the slot region remain relatively unchanged with respect to the locations. It is pointed out that the tangential electric fields $E_{z1}$ and $E_{z2}$ along the edge located at x=c give rise to the large edge current densities $J_{z1}$ and $J_{z2}$, respectively.

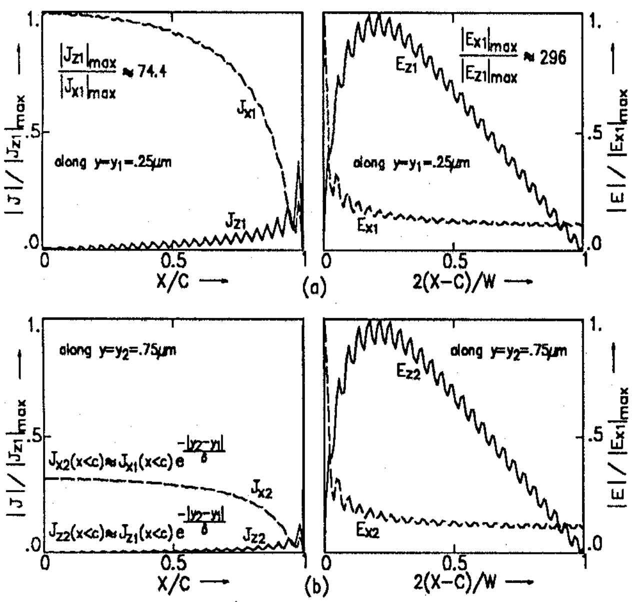


Fig. 2 The slot-fields ($E_x$ and $E_z$) in air region ($\varepsilon_a$) and current densities ($J_x$ and $J_z$) in metal region ($\varepsilon_m$) versus normalized coordinates along the x direction, $\sigma=3.333\times10^7$ S/m, W/b=0.5, frequency = 40 GHz, and t=5 $\mu$m. (a)along y=0.25 $\mu$m. (b)along y=0.75 $\mu$m.

Fig. 3 plots the surface currents (dotted line), edge currents (dashed line), and the total composite currents (solid line), respectively, for a gold-plated finline. The data for the surface currents and the edge currents are taken near x=c and along y=0.25 $\mu$m. As expected for a finline with a good metal coating, the edge currents contributed by the air modes decay exponentially from the metal edge at x=c as (5) and (7) describe. Since the conductivity is high, the current densities at the edge of the metal strip are mainly contributed by the edge currents as shown in Fig. 3. The surface currents $J_{xs}$ and $J_{zs}$ indeed represent the bulk current distributions inside the metal, whereas the edge currents diminish quickly. This means the bulk currents inside the metal strip are dominated by the surface currents. (10) explains this finding, since the surface currents are the summation of the sinusoidal terms.

Changing the value of conductivity from $3.333\times10^7$ S/m to $1.\times10^6$ S/m, Fig. 4 plots the edge currents and the surface currents and maintains all the physical and material parameters identical to Fig. 3. While (5) and (7) still apply to the edge currents, the surface currents near the edge of the metal strip are no longer negligible. Thus the total currents near the metal edge deviate from the edge currents. As a consequence, the perturbation theory for the transmission line loss analysis can not be applied. By the classification of the total composite currents of the metal strip into edge currents and surface currents, the physical reasons why the perturbation theory won't work now become clear.

Fig. 5 plots the total currents at the metal edge, x=c, against various values of the conductivity. Based on (5) and (6), the magnitudes of $J_z$ and $J_y$ are indeed proportional to the square root of the conductivity. Using (7), the magnitude of $J_x$

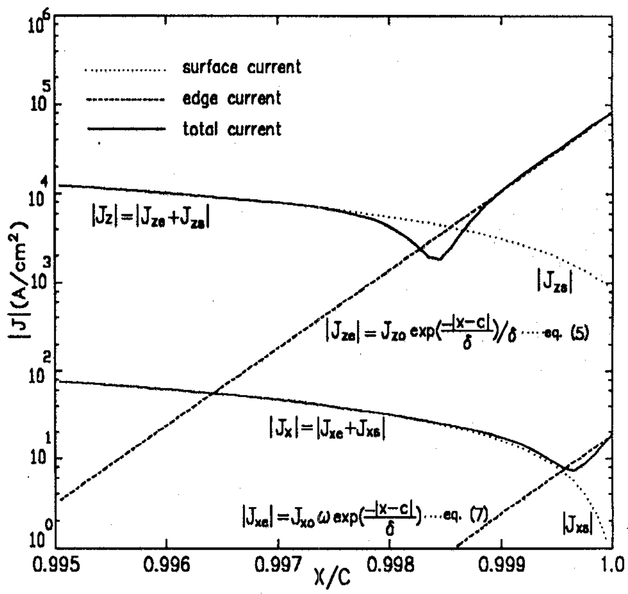


Fig. 3 The edge currents, surface currents, and total currents versus normalized coordinates near x=c along y=0.25 $\mu$m, $\sigma=3.333\times10^7$ S/m, W/b=0.5, frequency = 40 GHz, and t=5 $\mu$m.

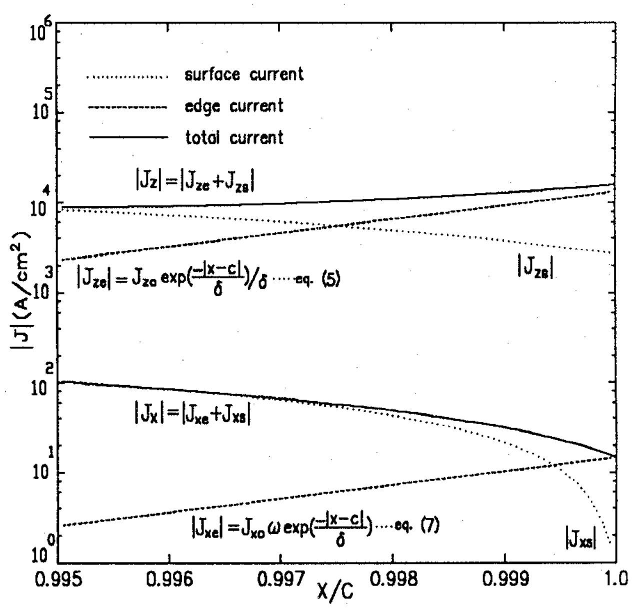


Fig. 4 The edge currents, surface currents, and total currents versus normalized coordinates near x=c along y=0.25 $\mu$m, $\sigma=1.\times10^6$ S/m, W/b=0.5, frequency = 40 GHz, and t=5 $\mu$m

should remain constant when the values of the conductivity vary. Indeed, $|J_x|$ is nearly constant. The same conclusions can be drawn by applying the perturbation theory [6]. Therefore any deviations from the straight lines obtained by the asymptotic expressions in Section II will imply that the perturbation theory starts to break down. In the particular case study shown in Fig. 5, the lower the value of the conductivity, the more deviations of the currents from straight lines. Therefore, the perturbation theory won't apply to the case of a metal strip with low conductivity.

In the limit of infinite conductivity; the skin depth vanishes, and it should be noted that the current densities $J_z$ and $J_y$ located at x=c become singular. It is also interesting to see that $J_x(x=c)$ keeps constant. This fact implies that the line of current flow is terminated by the time derivative of the charge on the surface[7]. Referring to the equation of continuity, the amplitude of surface charge density can be obtained and is approximately equal to $7.66\times10^{-11}$ C/cm$^2$ at the operating frequency of 40 GHz.

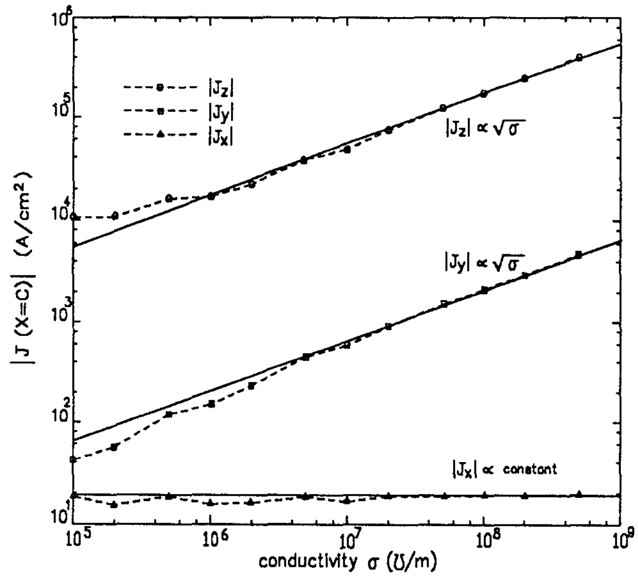


Fig. 5 The total currents located at the metal edge (x=c, and y=t/2) versus the conductivity of metal strips, W/b=0.5, frequency = 40 GHz, and t=2 $\mu$m.

## Conclusions

A full-wave approach to evaluate the current density on the metal strips of the unilateral finline is presented. The currents on the metal strips consist of the edge currents and surface currents contributed by the air and metal modes, respectively. In the particular case studied, this classification of the currents can derive asymptotic expressions for the skin-effect currents throughout the metal strips and enable us to know physically when the perturbation theory holds.

At a fixed operating frequency, the skin depth decreases with increasing conductivity. At infinite conductivity, the skin depth vanishes and the tangential current densities at the edge of the metal strip become singular. In contrast, the normal current density located at the edge is no longer zero and keeps constant, which is provided by the time derivative of charge on the surface.


## Acknowledgement

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